Uncertainty Propagation in a Computational Fatigue Model of an Airframe Structure

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Motivation: Why do we need Uncertainty Management?

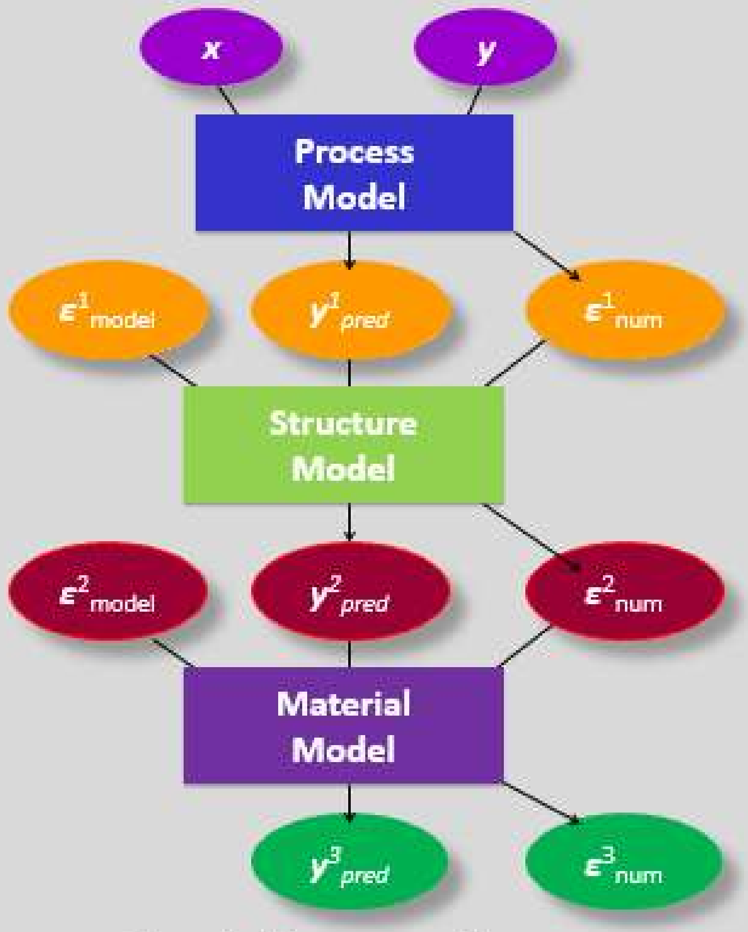
- Simulation-based design and certification is fundamentally about making decisions with uncertainty.
- The goal is to decide efficiently:
  - What is the actual uncertainty in the simulation results?
  - How will changing the scale and fidelity of the analysis impact the uncertainty in the results?
  - What does this mean for the product reliability?

Uncertainty exists : How do best manage its impact on reliability

Sources of Uncertainty & Propagation

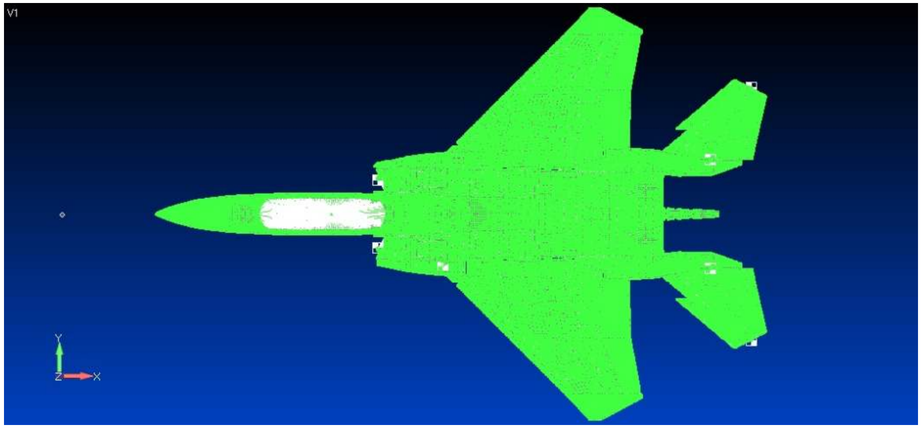
- Sources of Uncertainty :
  - Physical variability
  - Limited data
  - Statistical uncertainty

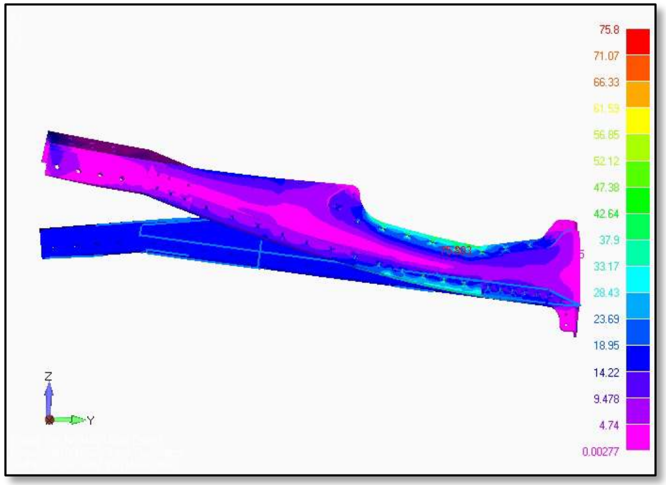
- Use All available data and knowledge
- Physics-based computational analysis
- Probabilistic analysis to explicitly propagate uncertainty
- Updating when new data/knowledge becomes available

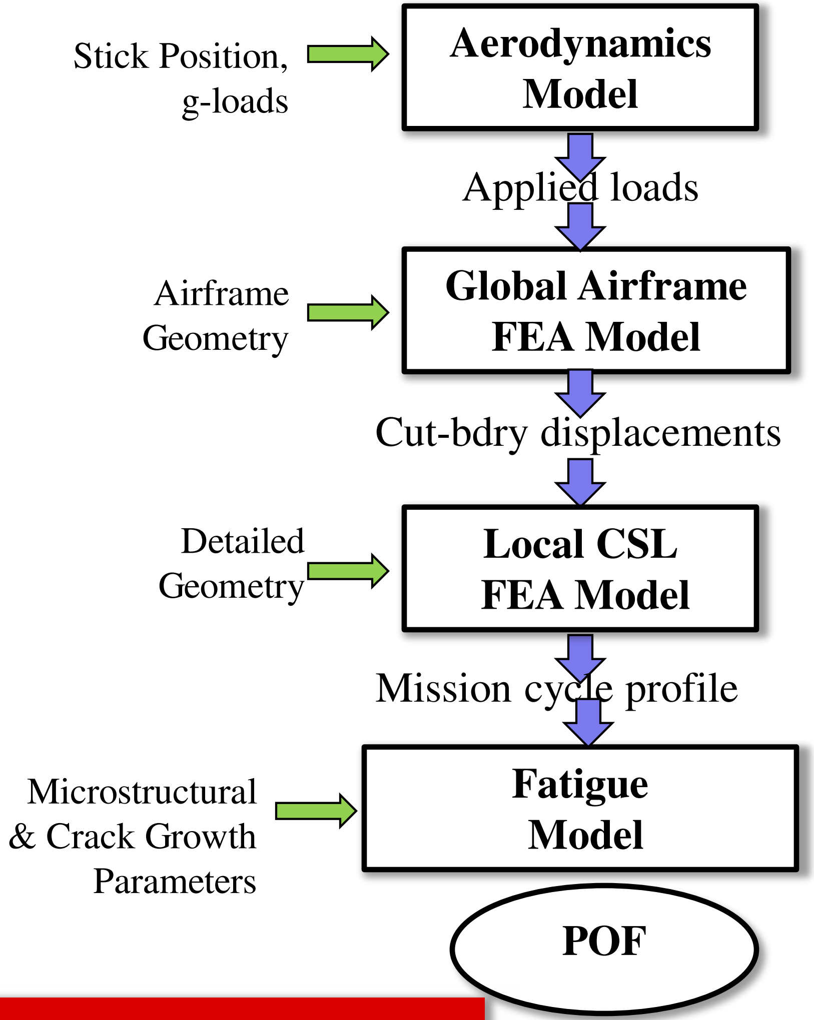


Uncertainty propagation across multiple levels in the Virtual Twin®

Multi-Disciplinary Example: Airframe Longeron







Stick Position, g-loads → **Aerodynamics Model**

Applied loads

Airframe Geometry → **Global Airframe FEA Model**

Cut-bdry displacements

Detailed Geometry → **Local CSL FEA Model**

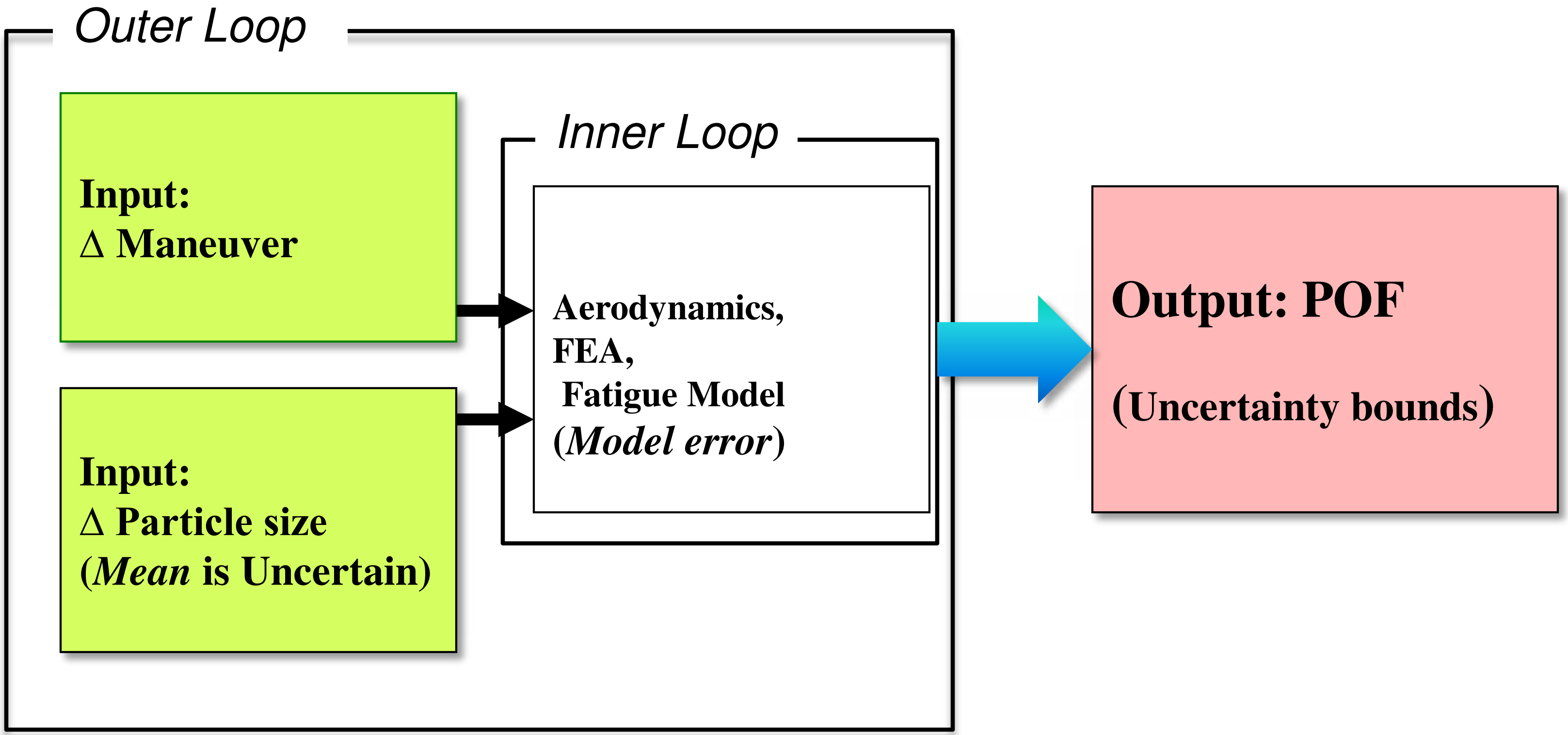
Mission cycle profile

Microstructural & Crack Growth Parameters → **Fatigue Model**

POF

What are the POF bounds due to uncertainty?

Monte Carlo Simulation Sequence



Outer Loop

Input:
Δ Maneuver

Input:
Δ Particle size
(*Mean* is Uncertain)

Inner Loop

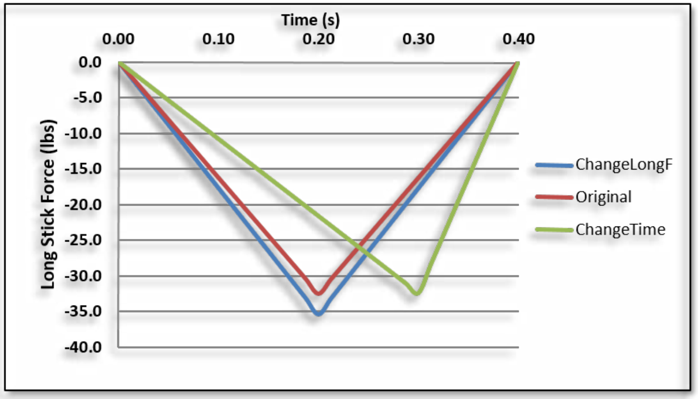
Aerodynamics,
FEA,
Fatigue Model
(*Model error*)

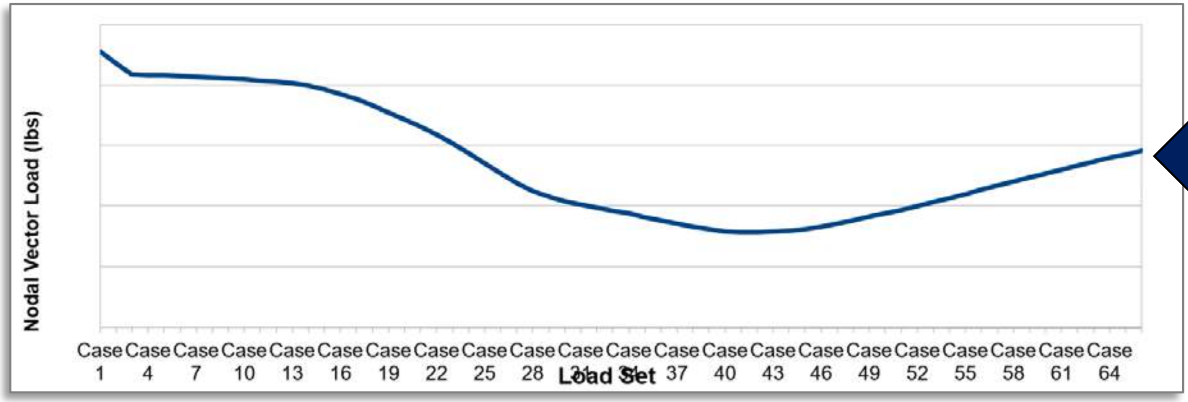
Output: POF

(Uncertainty bounds)

Aerodynamic Maneuver and Sources of Uncertainty

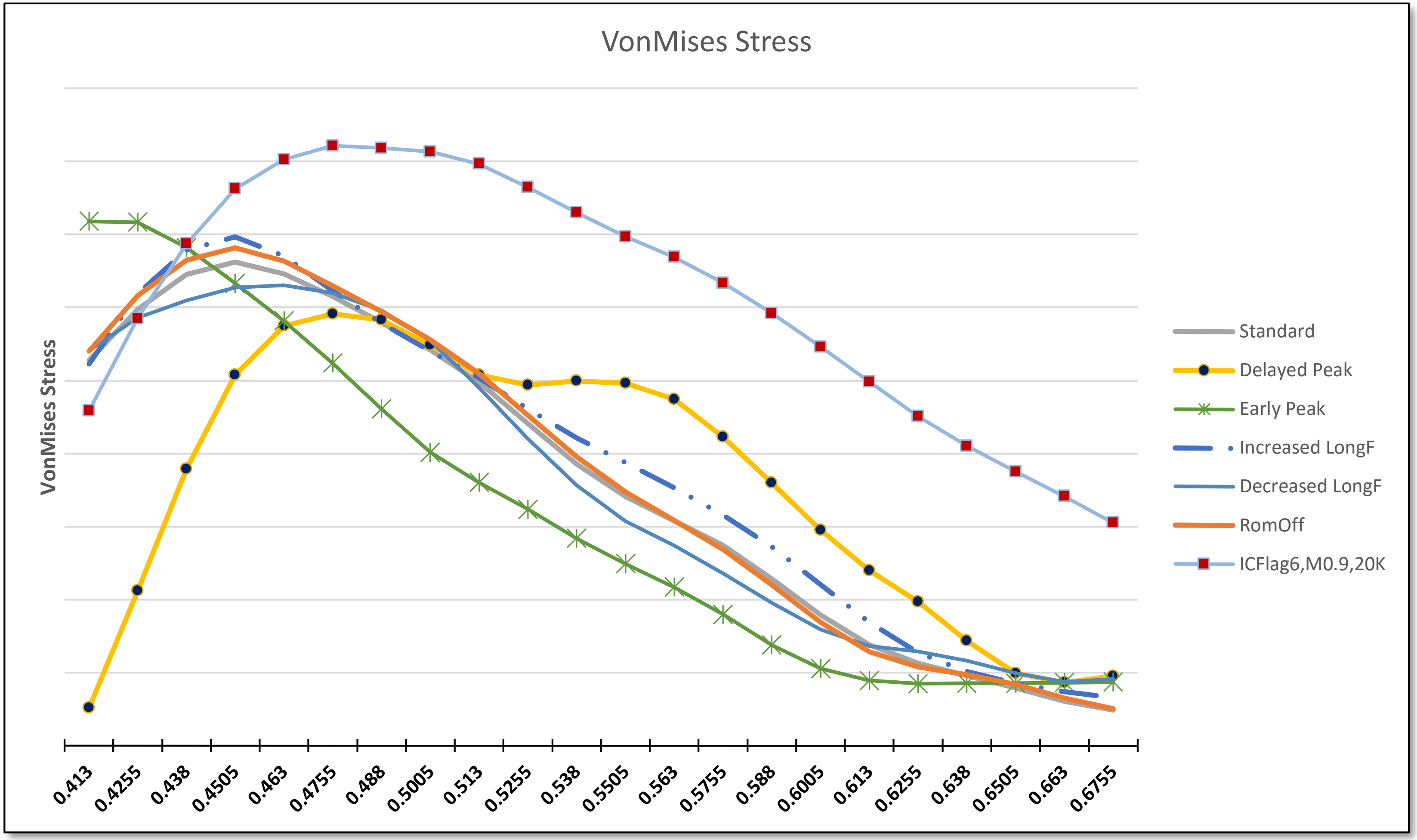
- Longitudinal Stick Force
- Peak time of Stick Force
- Mach #
- Altitude



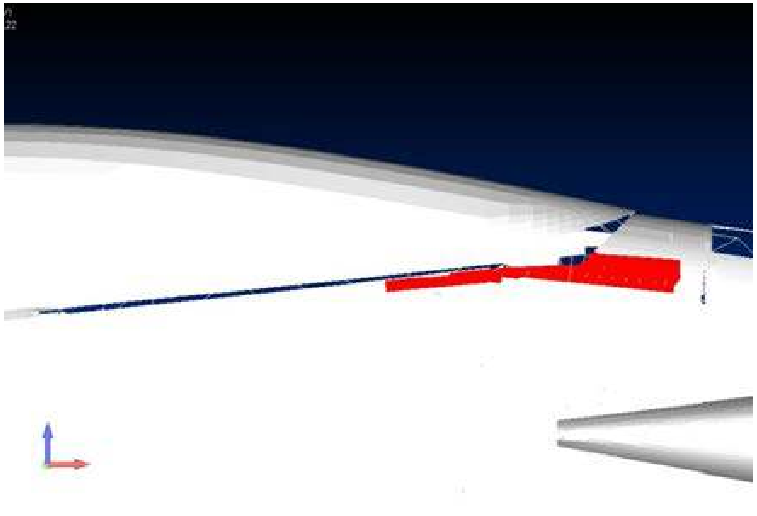


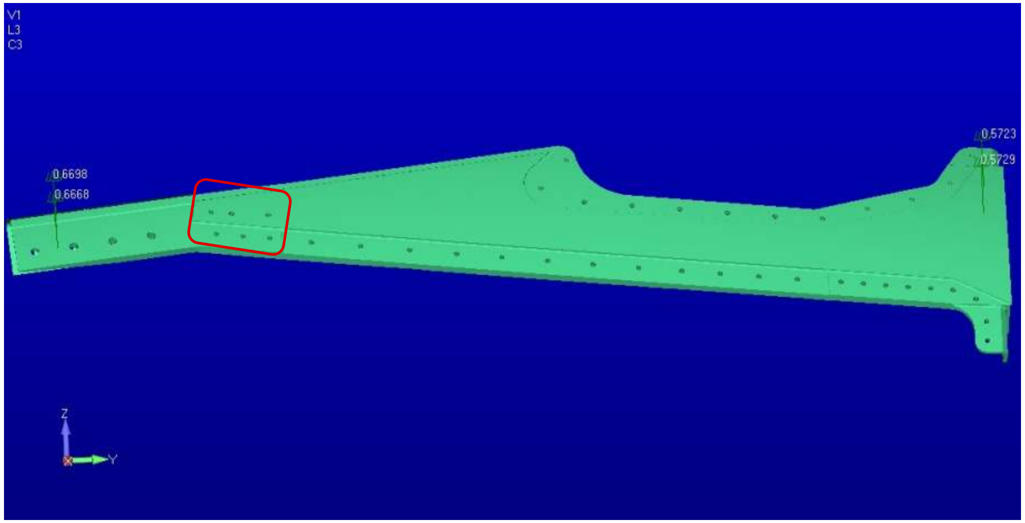
Global FEM with Maneuver loads

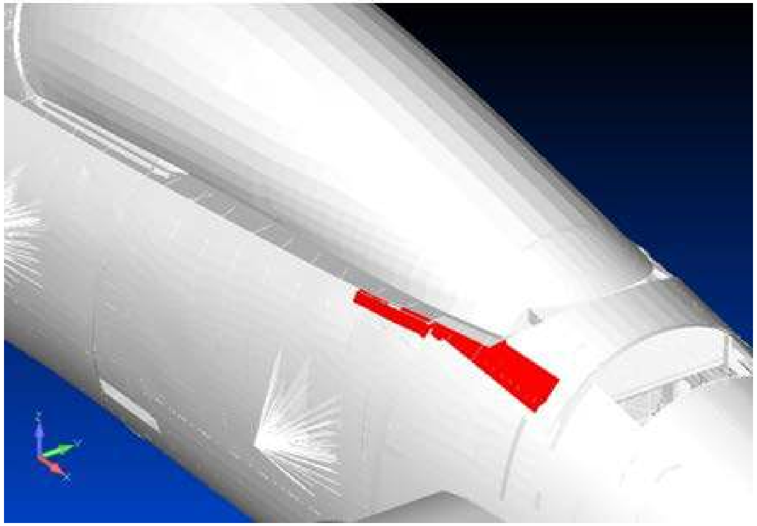
Uncertainty in Aerodynamics leads to Uncertainty in Global Stresses

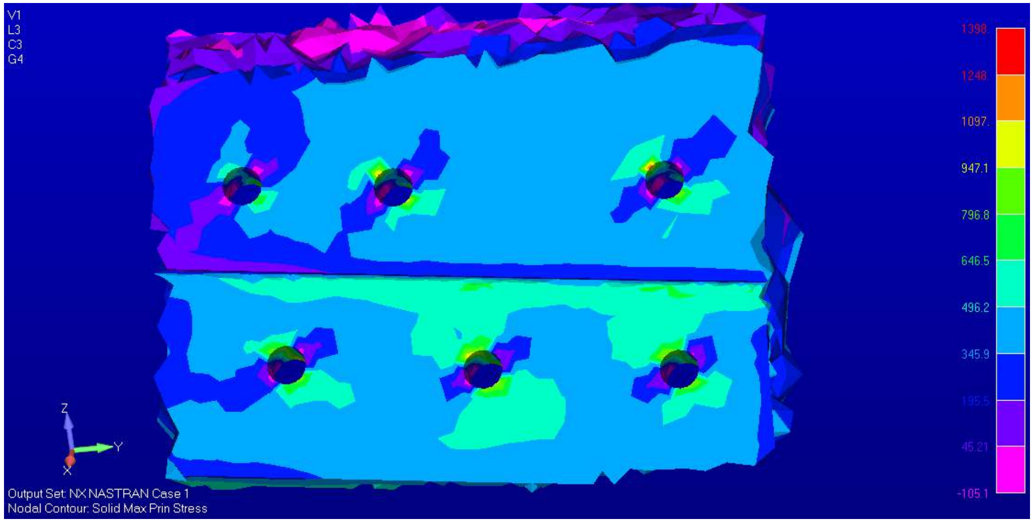


Orientation of the Local FEA

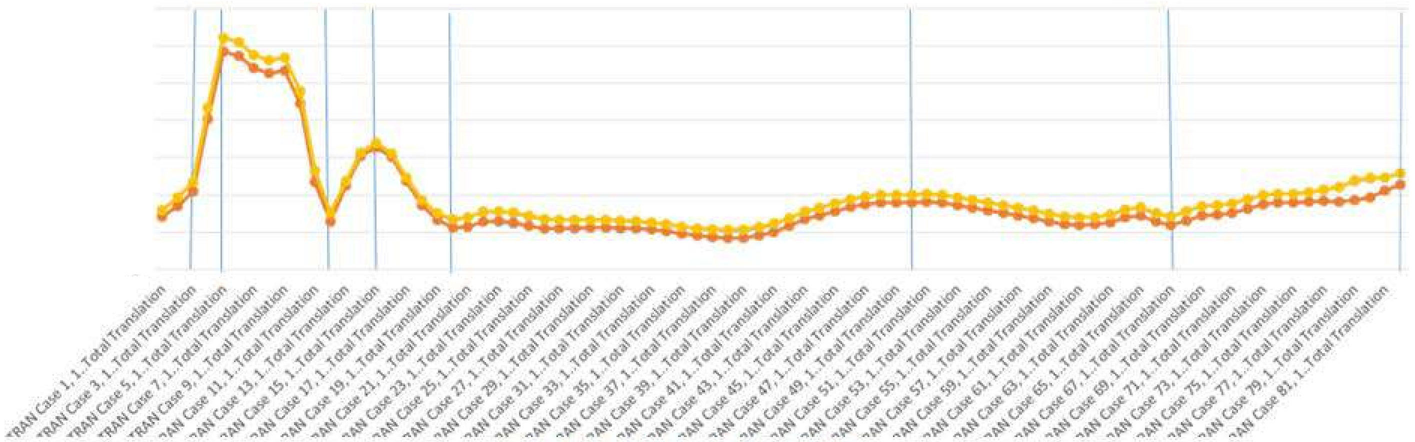




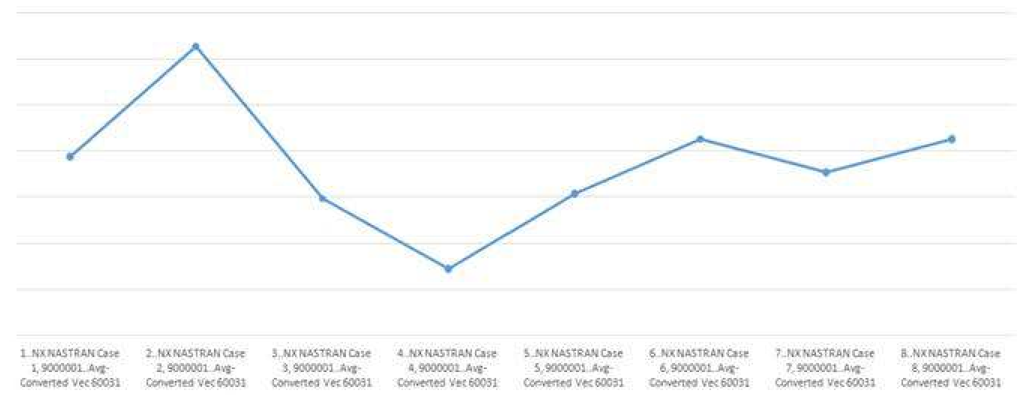




Filtered Mission Profile for a Single Maneuver

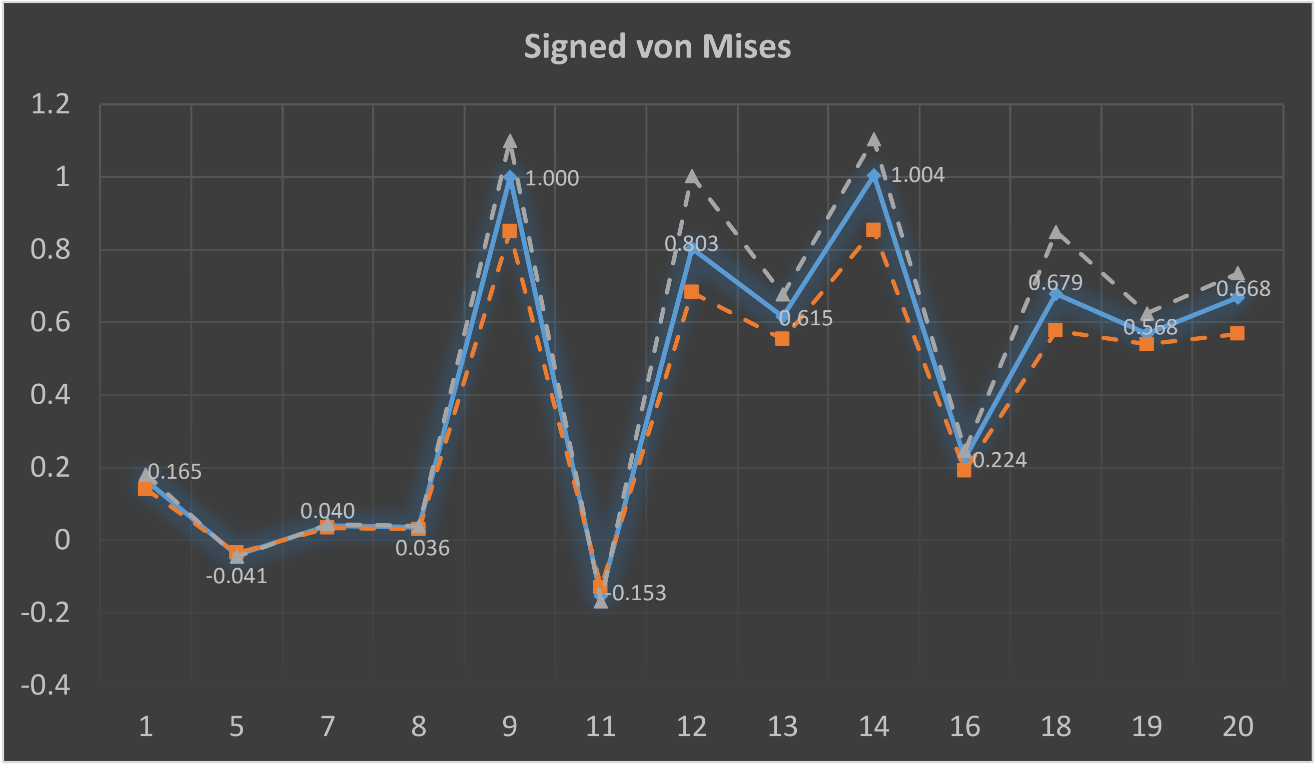


Global Filtered Spectrum



Local Filtered CSL Stress Spectrum

Final Mission Profile: Multiple Maneuvers, Scaled and Filtered



Final mission stresses are random

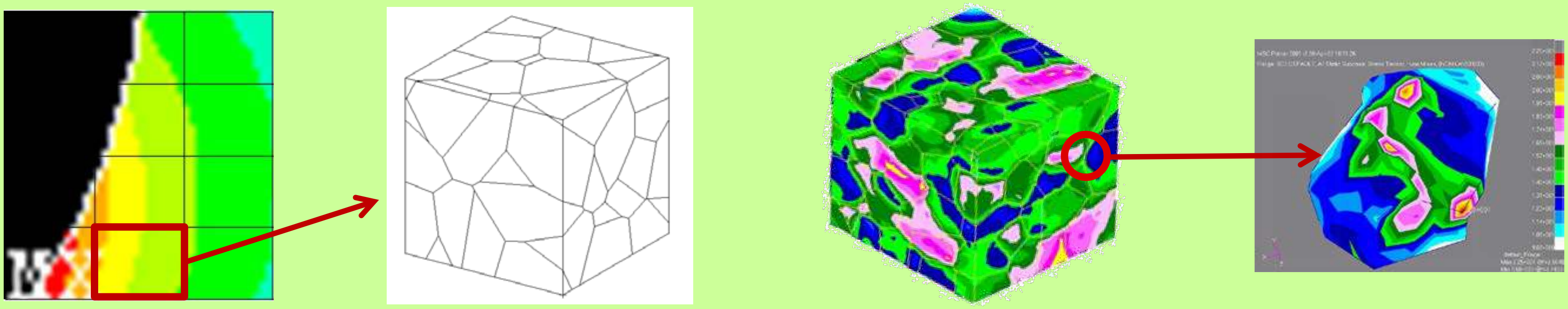
Final Mission

Hi	Lo	Repeats
0.668	-0.041	1
0.040	0.036	1
1.000	-0.153	1
0.803	0.615	1
1.004	0.224	1
0.679	0.568	1

Fatigue Analysis for 7XXX Aluminum

- Assumes damage starts at an inclusion
- If the damage can grow, progress to small flaw fracture mechanics (SFFM) and grow the damage for each cycle of the mission.
- Continue with SFFM until the average microstructural properties at the crack tip are equal to the bulk average material properties.
- If damage can still grow, progress to LEFM, Paris Law. Continue cycle-by-cycle damage growth until $\Delta K > \Delta K_{IC}$

Meshing FEA & Material

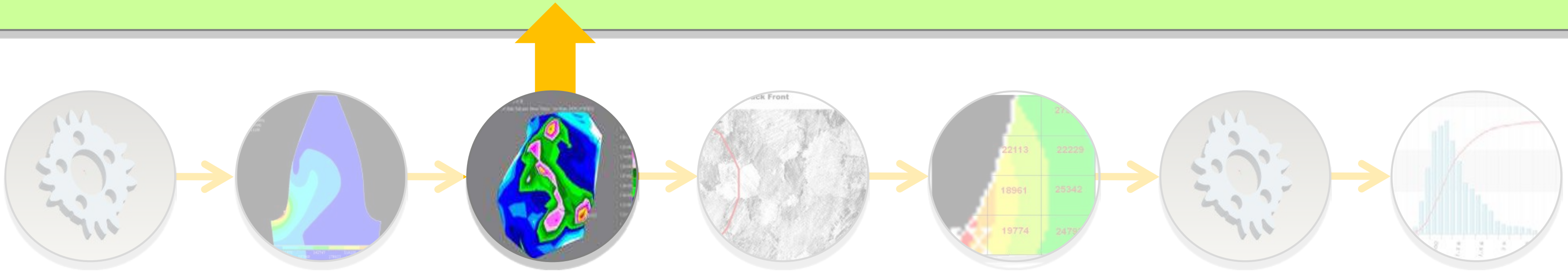


Initial Element

Superimpose Natural Substructure

Map FEA Stress to Substructure

Translate Microstress to each Grain



Component

Design Configuration

Material Configuration

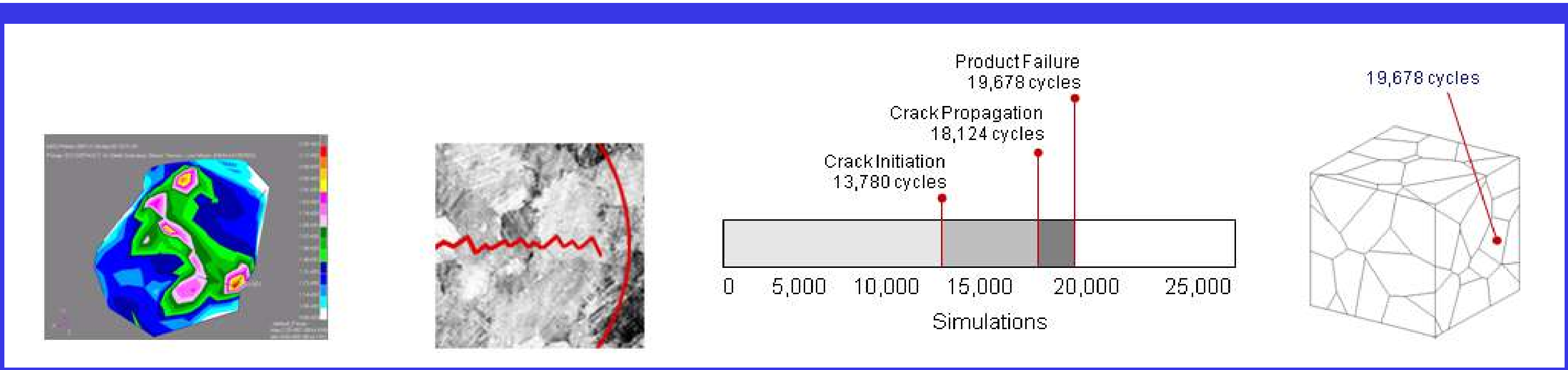
VLM Computational Processing

Mapping the Elements

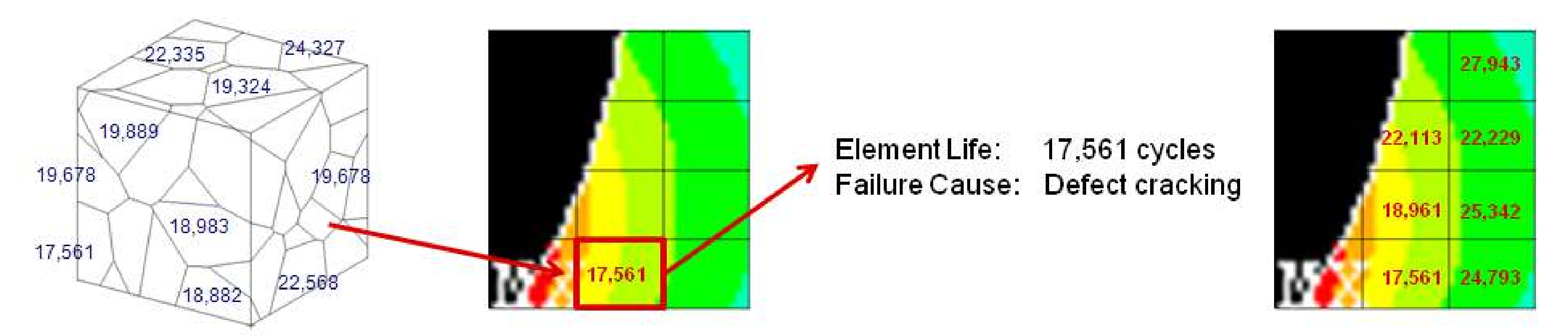
Component Simulation

FLEET Simulation

SFFM: Grain Level Processing

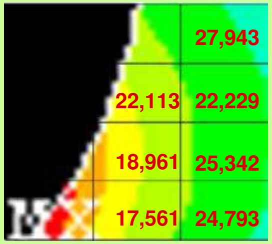


Grain characteristics and stress used in damage equations



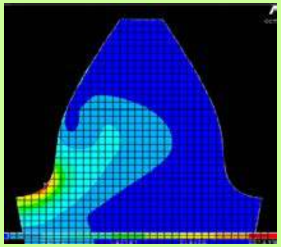
Grain lives statistically combined into element life and repeated for all FEA elements

Element – Component – System – Fleet

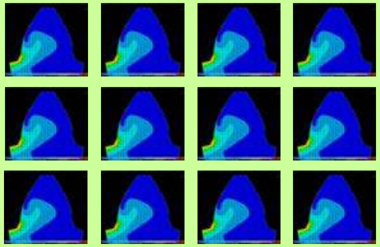


Integrate VLM Results with FEA

Tooth Life: 15,932 cycles
Failure Cause: Defects



VLM Integration for Entire Component

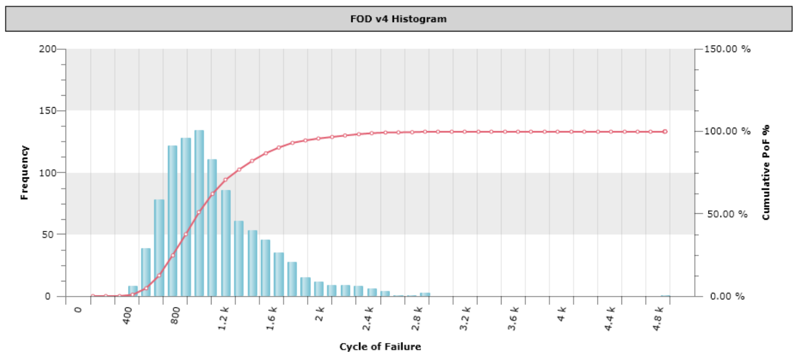


Repeat Sequence for Each Tooth

Component Life:
14,334 cycles

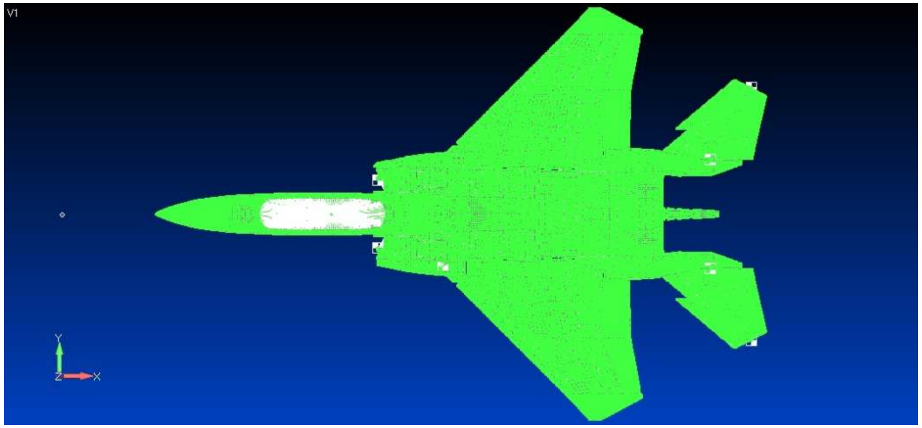
1st Virtual Twin Gear Simulated

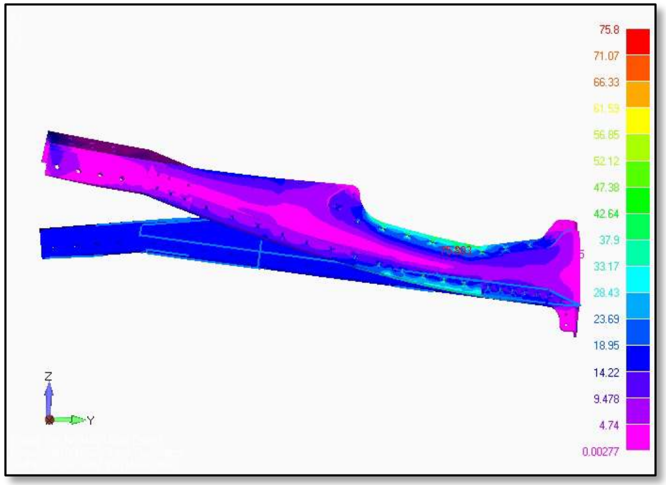
VT_1, VT_2, VT_3 … $VT_{1,000}$

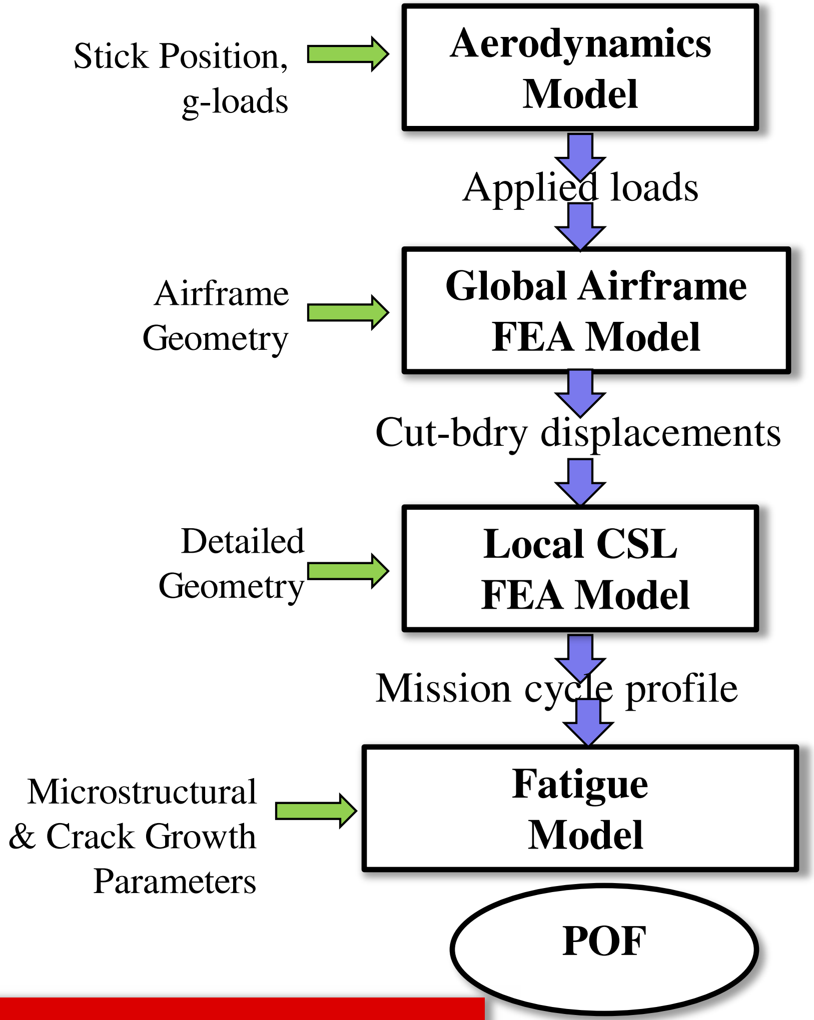


Run 1,000 Simulations

Multi-Disciplinary Example: Airframe Longeron







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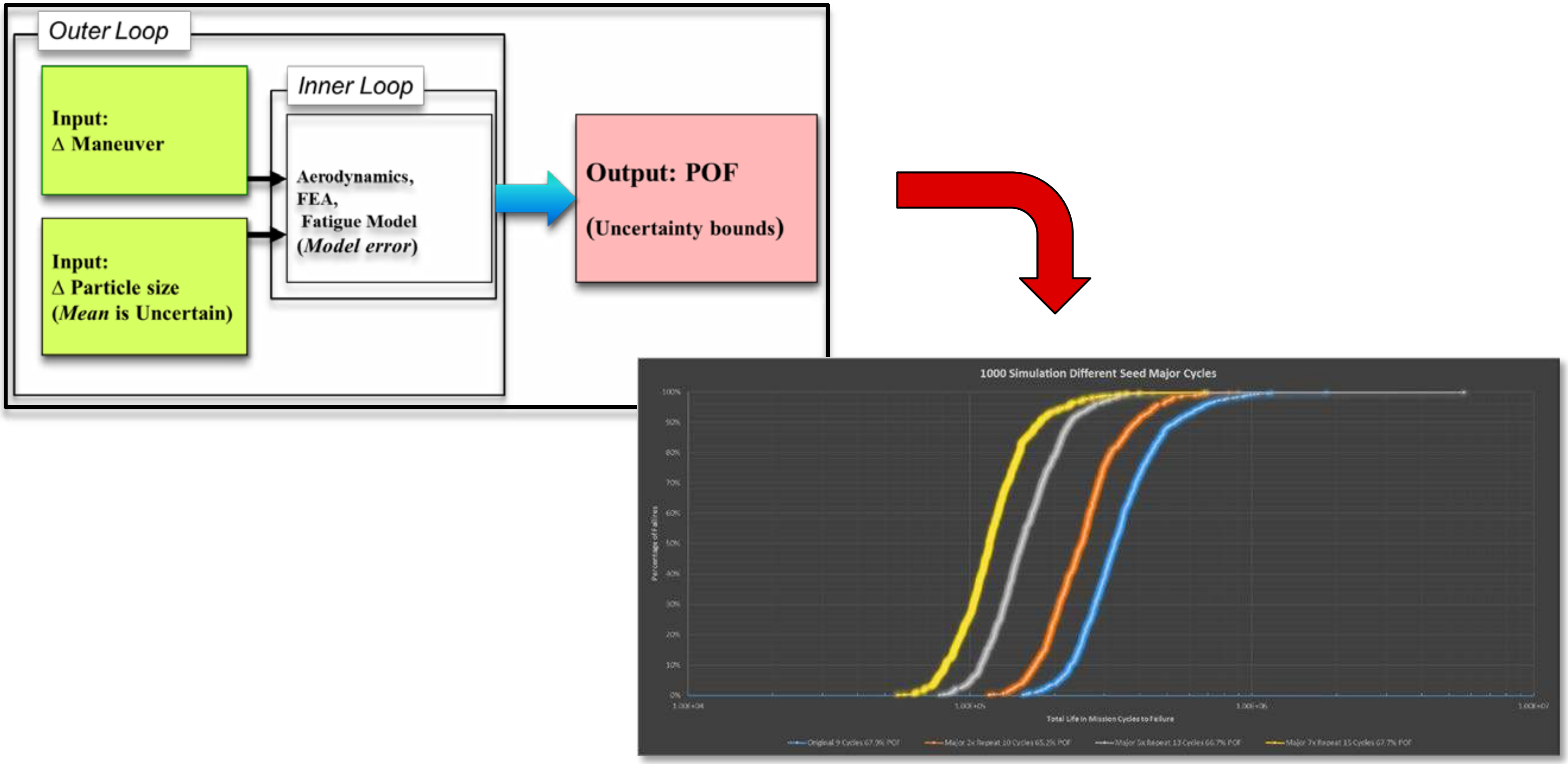
Inner & Outer Loop Simulation results



Outer Loop

Input:
Δ Maneuver

Inner Loop

Aerodynamics,
FEA,
Fatigue Model
(*Model error*)

Input:
Δ Particle size
(*Mean* is Uncertain)

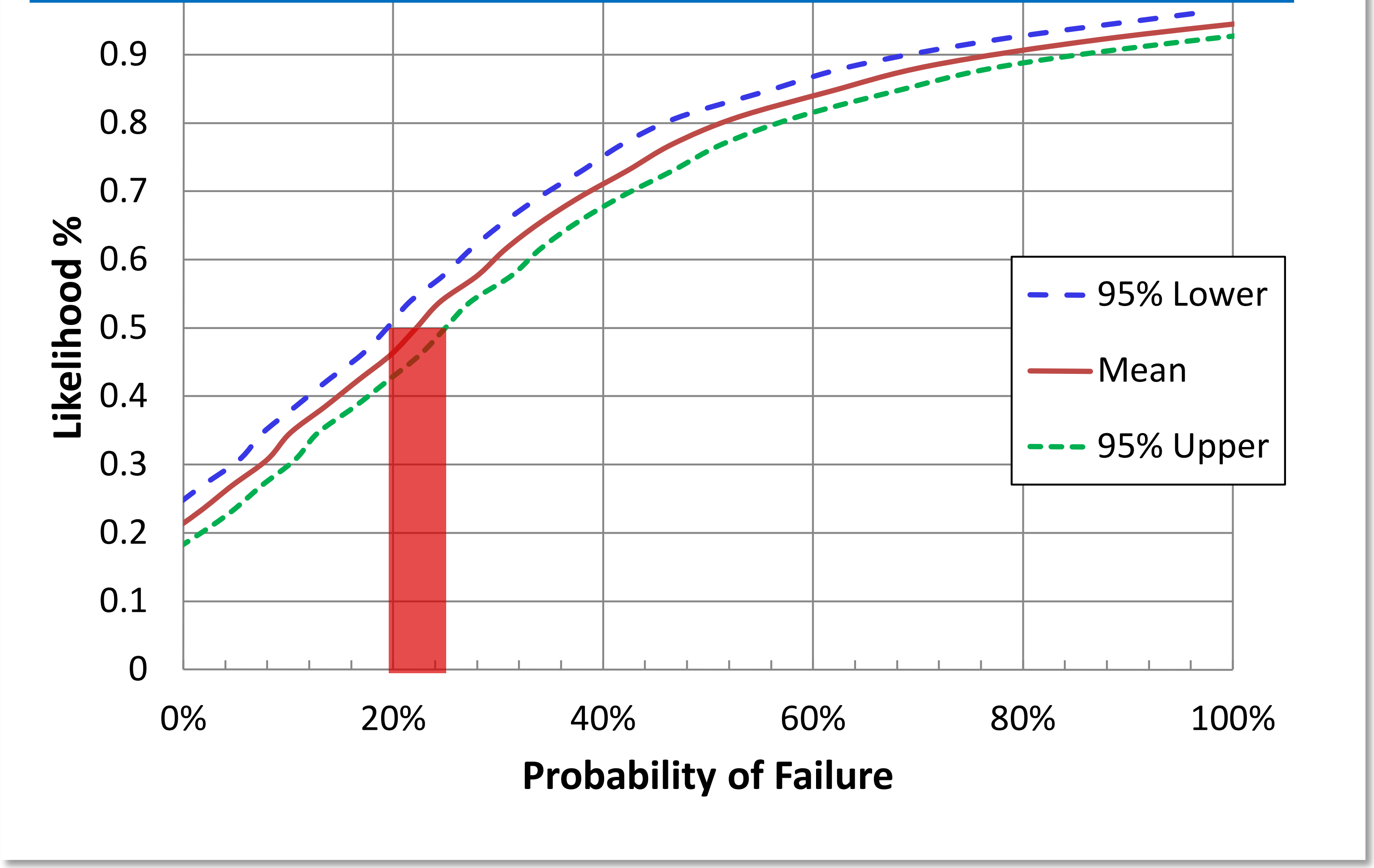
Output: POF

(Uncertainty bounds)

- POF itself becomes a random outcome

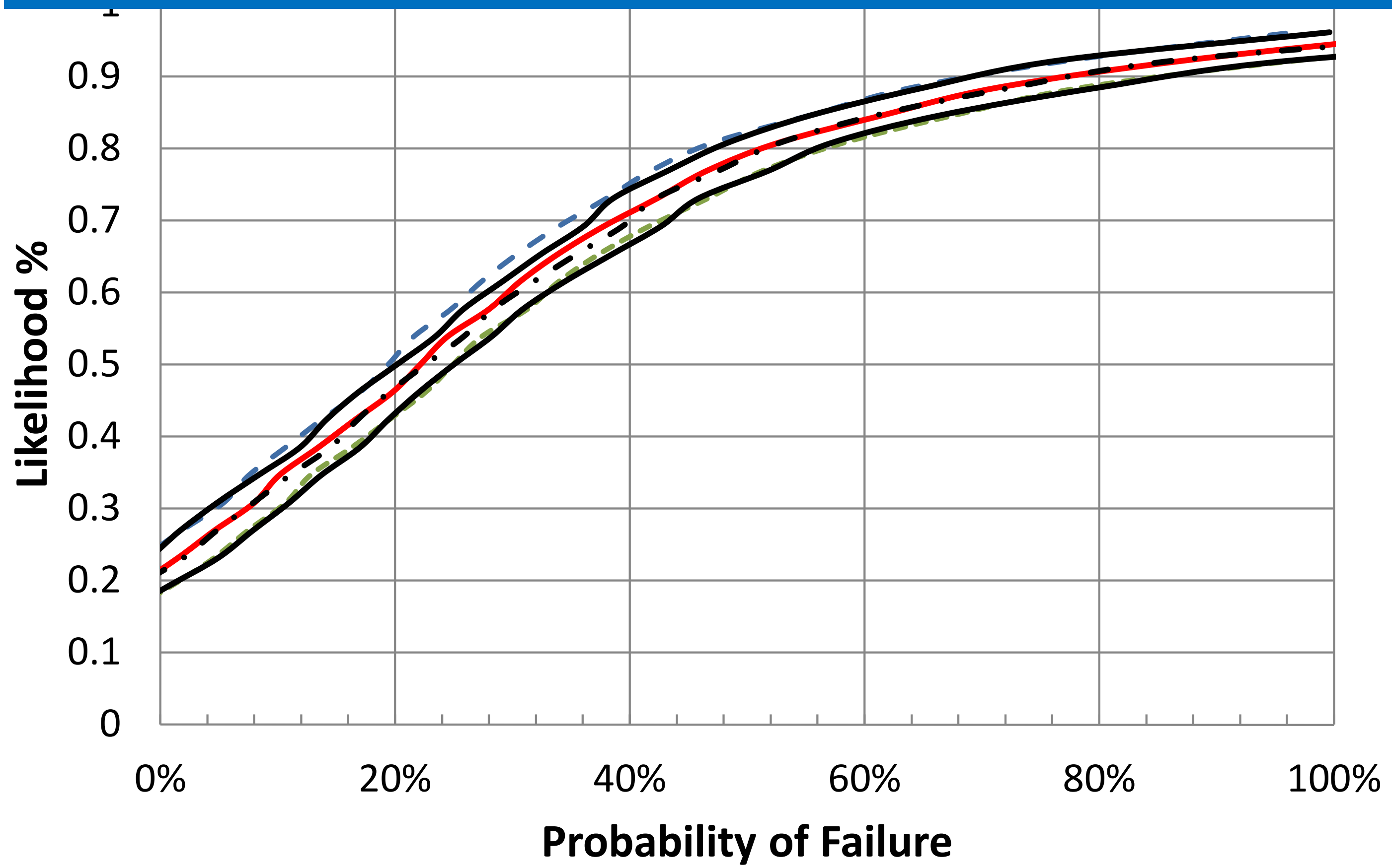
Result: Bounds on POF

Assuming there is no uncertainty in the RVs the median POF can vary between 20%-25%



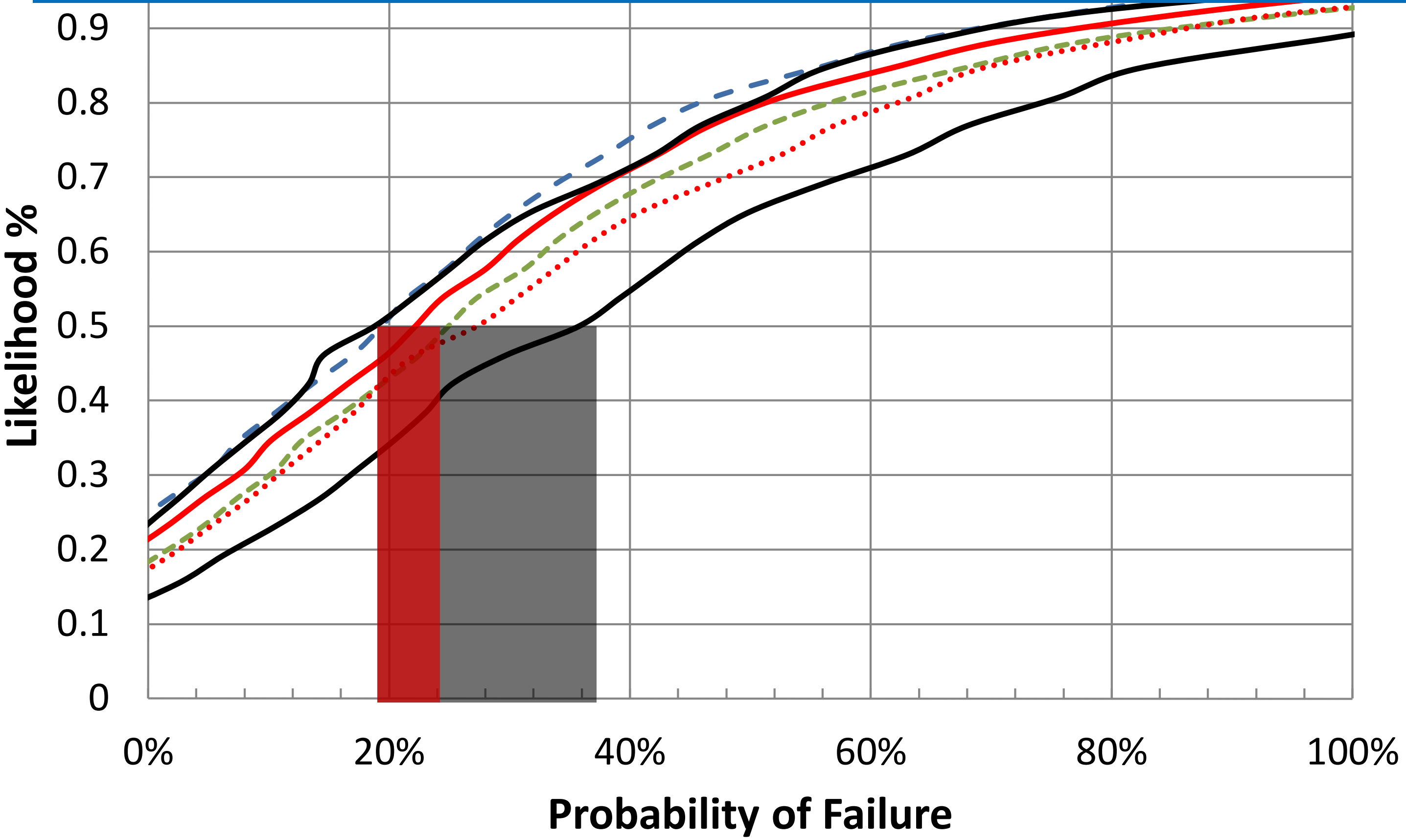
Sensitivity to Uncertainty in Mean Particle Size (50%)

Variation in particle size mean by as much as 50% shows *NO* noticeable change in median POF variation -20%-25%



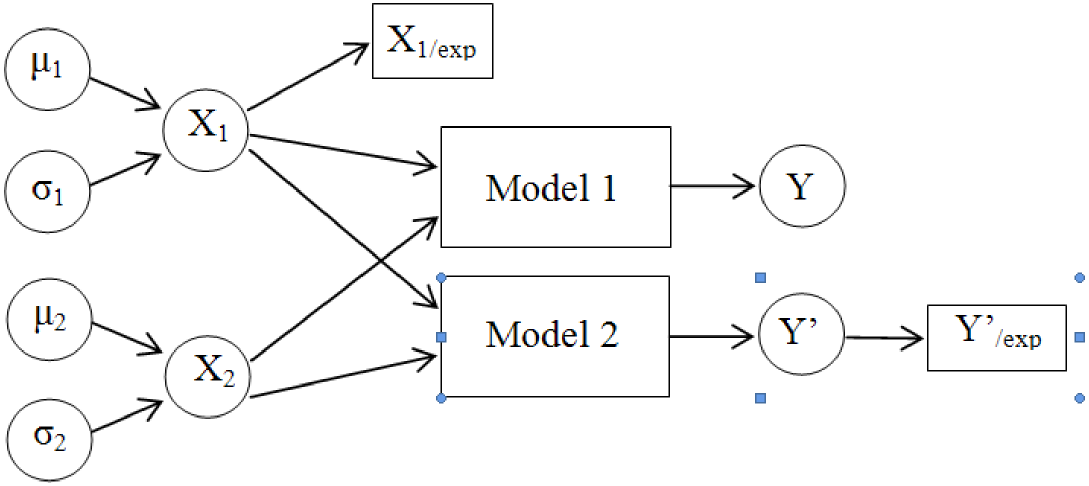
Sensitivity to Uncertainty in Mean Stress Amplitude (15%)

15% uncertainty in mean Δσ results in median POF increasing to 28% with bounds between 20% and 36%



Bayesian Update Methodology

It is a method to update the model parameters based on updated/experimental results



Bayes Network

- *X_1 is updated through experimental results of X_1 (Model 1)*
- *X_2 can be updated from experimental results of Y'(Model 2)*

- *Now updated X_1 and X_2 can be used to update the result Y*

Computation of Posterior Distribution

- Prior density= $f(X_1)$
- Data = $X_{1/exp}$ (i.e., $X_{1/exp}$ is an observed value of X_1)
- Posterior density = likelihood of X_1 given $X_{1/exp}$ is observed = $f'(X_1)$

$$f'(X_1) = \frac{f(X_1)f(X_{1/exp}|X_1)}{\int_{-\infty}^{\infty} f(X_1)f(X_{1/exp}|X_1)}$$

Methods such as Metropolis-Hastings (M-H) and slice sampling algorithms are very useful in generating posterior density functions

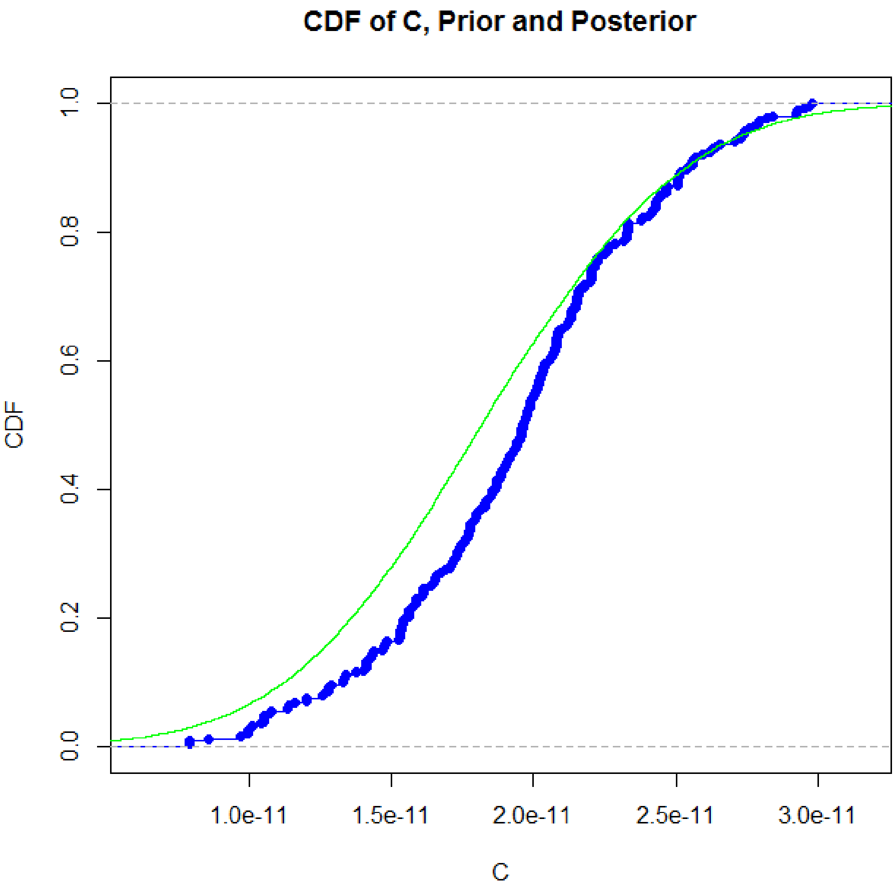
Problem

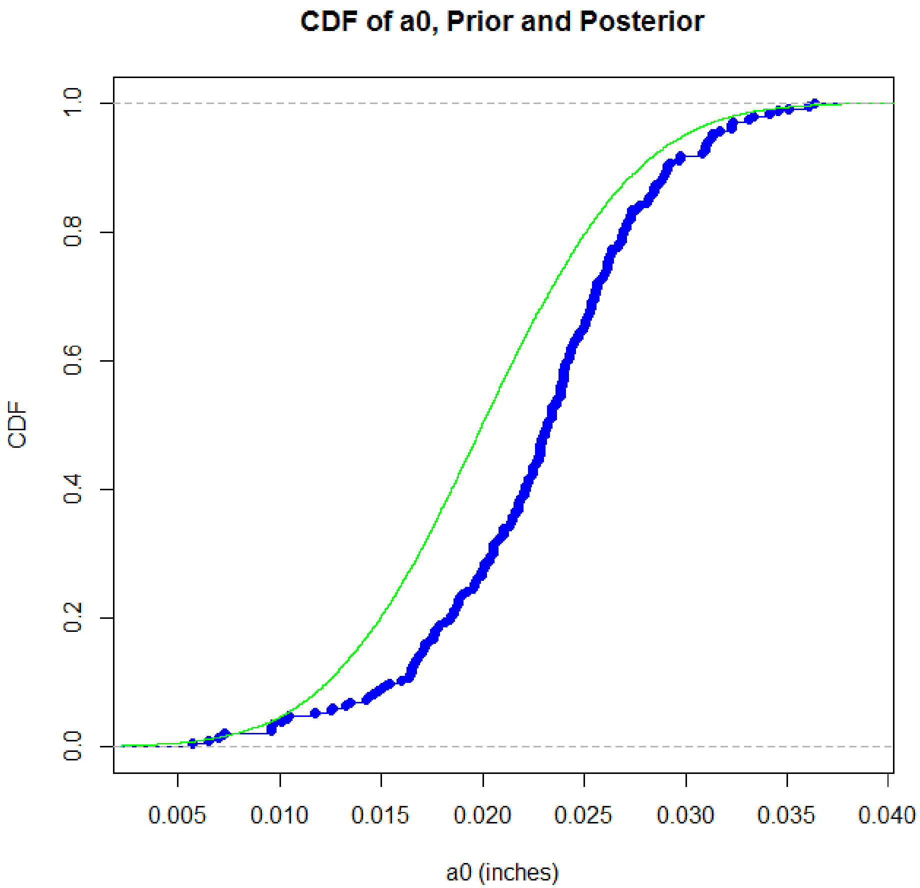
- Cracks grow according to the following equation:
  - $a_m = a_{m-1} + C * (\beta * \sqrt{\pi})^n \left(\sqrt{a_{m-1}}\right)^n (\Delta\sigma_1{}^n + \Delta\sigma_2{}^n + \Delta\sigma_3{}^n + \cdots)$
  - a_{m-1} is the crack size at the previous mission
  - C is Paris Law Coefficient
  - β = 1.12 is the crack shape parameter
  - n = 4.73 is Paris Law exponent
  - $\sum_{i=1} \Delta\sigma_i{}^n = 69243532$ is the mission profile
  - The equation is calculated recursively to obtain the crack size after “m” missions
- Given observations of the crack at a specific mission, we want to use a Bayesian model to update the crack size (a0) and & Paris Coefficient(C) at mission 0

Bayesian Experiment - 3

- Posterior Distribution
  - Single Observation of 1 inch: (Larger than expected median value)
    - C = (1.944 e-11, 0.24)
    - a0 = (0.0227, 0.25)

Bayesian Experiment-3 results


CDF of C, Prior and Posterior


CDF of a0, Prior and Posterior

As expected the posterior (blue trace) shifts to the right since the observation was larger than expected value

Benefits of Uncertainty Propagation Model

- Sensitivity of the uncertainty in the analysis prediction to each uncertainty/approximation can be estimated
- Once a Computational Model is built, the methodology can be used to continually update based on new information to arrive at most robust predictions.
- A more judicious allocation of computational fidelity and resources can be made without sacrificing accuracy.

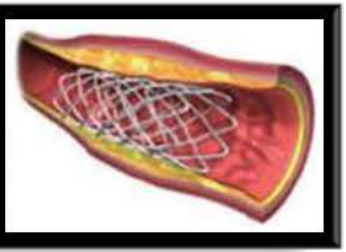
VEXTEC U.S. Federal Govt. Relationship

VEXTEC Clients	Successes Achieved With VEXTEC Technology
USAF	• Partner in the Airframe "*Digital Twin" Initiative* • Use UQ/UM to calibrate and predict test results • Predict the confidence bounds on damage risk
FDA	• Only 1 of 2 companies in the *MDDT Pilot program* • Use VLM to simulate and certify bench testing of cardiac leads
USN	• Use VLM +UQ/UM to forecast fleet maintenance • Predict Fatigue + Corrosion Damage risk


Aircraft Parts


Automotive


Industrial Equip


Medical Implants

Over 100 VEXTEC Commercial Successes

VEXTEC Clients	Successes Achieved With VLM
American Airlines	$4 M/yr saved on bearings
Cummins Engine	$5 M saved from $150K investment
Boston Scientific	Working with FDA towards methods approval
Oil & Gas Co.	$12 M /yr saved in equipment leasing
Fortune 500 Co.	$3 M saved in manufacturing line maintenance
Fortune 100 Co.	$250 K/month on machining efficiencies
Chrysler	Early Adopter using VEXTEC software since 2001

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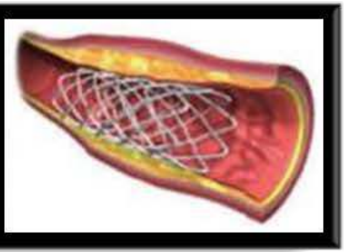
Aircraft Parts

Automotive

Industrial Equip



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Thank You!

- Founded in 2000 in Nashville
- Software backed by 7 Patents: Virtual Life Management® (VLM®) & VPS-MICRO®
- Value Proposition: Help companies assure product reliability and reduce cost
  - Leverage physical testing for increased confidence
  - Forecast product durability and manage product life cycle risk
- Business Model: Hybrid – consulting services, software licensing and training

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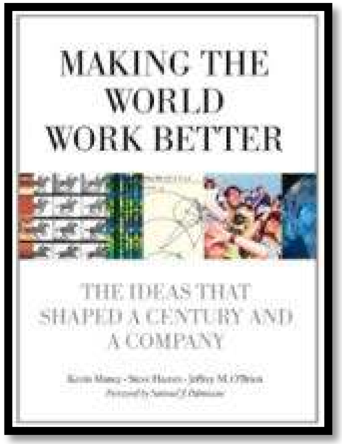

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