ABSTRACT

This paper discusses the simulation of creep fatigue damage incorporating damage mechanisms based on the underlying physics of failure. Computational models are used to predict the material response by explicitly modeling the randomness of the material microstructure, interacting loading, dwell and temperature effects. The variation in the material response is determined using mesomechanical models applied at the grain and sub grain level. The model predictions were compared with smooth specimen laboratory test data for nickel-based alloys from 1200°F to 1350°F. The model does an excellent job of capturing the effect of temperature on the fatigue life of the specimens. Finite element analysis (FEA) was performed for notch specimens. The results of an elastic-plastic analysis and peak dwell creep are taken and used to simulate the load conditions for a notched specimen at various temperatures and different maximum applied stresses. The model correlates very well with the notched specimen test data and captures the variability that would be expected in the test.

INTRODUCTION

Obtaining dwell fatigue life prediction models through physical testing on a new material system is a daunting, time-consuming, expensive and practically open-ended task. It is common for a normal fatigue test to operate at ten cycles per minute. Dwell testing requires cycles that may be 100 times longer at elevated temperatures, which costs time and resources. Traditional statistical life prediction testing can require hundreds of tests, as with some aerospace OEMs. A significant reduction in manpower, machine time, and capital is realized immediately with a physics based prognosis capability.

Fatigue can be viewed as an entire range of damage accumulation sequences from crack nucleation of the initially unflawed structure to final fast fracture. Current life prediction methods for metallic components consider three stages of fatigue: crack initiation, long crack propagation and final fast fracture. The crack initiation stage can be broken down into crack nucleation and small crack growth stages. Long crack growth stage can be modeled by using linear elastic or elastic plastic fracture mechanics. However small crack growth stage shows crack growth rates that deviates significantly from the rates predicted by conventional fracture mechanics [1]. Crack nucleation has been shown to occur on the order of the grain size. Each of the nucleation, small crack growth and long crack growth stages must be modeled separately in physics-based predictions since different mechanisms control the damage accumulation at each stage. In case of low cycle fatigue, where the loading is relatively high, crack nucleation takes place early and most of the fatigue life is spent in crack growth stages. However, high strength materials may spend most of the fatigue life in crack nucleation stage even for low cycle fatigue [2]. This shows the relative importance of modeling crack nucleation stage in predicting fatigue life of a component.

All three stages of damage also show significant scatter. Ishii et al. found that for low cycle fatigue conditions for NiCrMoV steel, the scatter in crack nucleation life can be as much as 44.2% [3]. To accurately predict the reliability of a component, the scatter in fatigue life must be accounted for.

Addition of a tensile dwell period during fatigue cycling can lead to greater damage and lower lives in some engineering alloys [4]. The deformation and failure mechanisms are different for different alloys depending on test temperatures and material properties [5]. With the application of tensile dwell, stress relaxation can occur, lowering the material’s strength in that loading direction. Cyclic softening can occur with a combination of low strain rate, long dwell period, low stress range, and large relaxation strain (which is a result of transition from elastic to plastic strain) that can promote intergranular damage. Damage features such as cavity formation and grain boundary sliding result in faster failure than might occur with transgranular damage. Other processes such as environmental interactions by oxidation can accelerate the growth of damage [5]. Physical models of these damage mechanisms are combined with computational model of the microstructural geometry to simulate damage accumulation and growth.

SIMULATION MODEL DESCRIPTION

Following the sequence of damage mechanisms, the simulation model consists of three stages – crack nucleation, small crack growth and long crack growth.
CRACK NUCLEATION

The crack nucleation mechanism can be either transgranular, intergranular or both, depending on the applied temperature and stress. The objective is to calculate the number of cycles to nucleation $N_n$. In addition to nucleation from a grain, cracks can also form at the sites of inclusions and pores. Nonmetallic inclusions (NMIs) have a different coefficient of thermal expansion from the matrix material. Thermal strains accumulate during cooling phase of material heat treatment inducing residual stresses and local plastic yielding near the inclusions. Pores provide a local stress concentration. These sites can cause a crack to nucleate if they are sufficiently large.

Transgranular nucleation without dwell

Tanaka and Mura [6] have proposed a microstructural crack nucleation model to predict the nucleated crack size and life. The model predicts damage accumulation through irreversible dislocation pile-up at microstructural obstacles. Cracks nucleate when critical strain energy is exceeded for the plane where dislocation pile-up occurs. The model is also consistent with Coffin-Manson relationship for fatigue crack initiation, the Hall-Petch relationship for grain size dependency of crack initiation life and the Palmgren-Miner law of damage accumulation.

In this model the crack nucleation life is given as

$$N_n = \frac{4GW_c}{(\Delta\tau - 2k)^2\pi(1-\nu) d}$$  \hspace{1cm} (1)

Here $G$ is the shear modulus of the material, $W_c$ is the critical value of strain energy required to nucleate a crack, $\Delta\tau$ is the shear stress range on the plane of dislocation pile-up, $k$ is the frictional stress and $d$ is the grain size. This model assumes that

1. The grain is homogeneous.
2. Damage accumulates on a single planar system.
3. Crack nucleation size is equal to the grain size.
4. Dislocation movement is irreversible and the dipoles pile up monotonically at the grain boundaries.
5. The number of cycles to saturation is negligible.

Transgranular Nucleation with dwell

The model for transgranular nucleation can be modified to include the effects of dwell by considering the plastic work done during the tensile hold period [7]. This additional work can be shown to be given by the relation

$$W_c = \frac{\sigma_{max}^2 - \sigma_{s}^2}{2E}$$  \hspace{1cm} (2)

From the creep strain rate and stress relation, the relaxed stress $\tau_r$ work can be expressed wholly in terms of the applied maximum stress $\tau_{max}$. The modified crack nucleation life for fatigue cycling along with a tensile dwell period then becomes

$$N_n = \frac{W_s}{w_r + W_s}$$  \hspace{1cm} (3)

If the critical strain energy for crack nucleation is different under creep and fatigue, Skelton [7] proposed a modified model to calculate the nucleation life. In this model the crack nucleation life is given by

$$N_n = \frac{1}{\left[ \frac{W_c}{W_r} \right]}$$  \hspace{1cm} (4)

Here $W_r$ is the critical strain energy for nucleation in pure fatigue and $W_c$ is the critical strain energy for crack nucleation in pure creep. When the critical energies are equal, this relation reduces to Eq.(3).

Intergranular crack nucleation

Mura and Tanaka [8] proposed an intergranular crack nucleation model for initiation of cracks along grain boundaries. They modeled the damage as accumulation of surface dislocations along the grain boundary. For a circular grain of diameter $d$, the crack nucleation life is given as

$$N_n = \frac{4GW_{W}}{(\Delta\tau - 2k)^2 d}$$  \hspace{1cm} (5)

Here $W_W$ is the critical strain energy required to nucleate an intergranular crack. Notice that this model is very similar to the transgranular crack nucleation model in Eq. (1).

SMALL CRACK GROWTH

A small crack can be thought of as a crack with a size on the order of the microstructure. As mentioned earlier, the behavior of small cracks can be quite different from the behavior of long cracks. While long crack behavior can be predicted using conventional fracture mechanics techniques, small crack growth rates can vary by orders of magnitude from predicted fracture mechanics crack growth rates. This is due to two competing factors: higher growth rates due to lack of closure and growth retardation due to microstructural obstacles. Planar growth of small cracks causes the crack surface to be smoother than long cracks. Small cracks are more likely to be open at zero load [9].

Crack opening displacement (COD) has been experimentally observed to be correlated with varying small crack growth rates [10]. In particular the small crack growth rate is found to vary almost linearly with the crack tip opening displacement (CTOD) for aluminum, nickel and titanium alloys [11]. The relation between these two quantities can be expressed as

$$\frac{da}{dN} = C(\Delta\phi)$$  \hspace{1cm} (6)

Here $C$ is the small crack growth coefficient and $\Delta\phi$ is the range of CTOD at the crack tip between maximum
and minimum load. Addition of a dwell period will change the crack growth rate along with a corresponding change in the CTOD. The CTOD can be related to the stress intensity factor by Irwin’s analysis [12] as

\[ \phi = \frac{4}{\pi} \frac{K}{\sigma_y L} \]  

(7)

Riedel and Rice [13] introduced the C(t) parameter to estimate the change in stress and strain fields in front of a crack tip due to creep. This parameter is related to the stress intensity factor and thus can be related to the CTOD. The use of CTOD which changes with time has been used as a creep crack growth driving force parameter elsewhere in the literature [14].

The contributions to total crack growth during a cycle from fatigue cycling and creep during the hold period can be assumed to be related as

\[ \left( \frac{da}{dN} \right)_\text{total} = \left( \frac{da}{dN} \right)_f + \left( \frac{da}{dt} \right)_\text{hold} \]  

(8)

The driving mechanism for creep crack growth is assumed to be grain boundary sliding. The creep crack growth rate can be related to a creep process zone of length L, void spacing \( \lambda \), grain size d and the average grain boundary sliding strain rate as [15]

\[ \frac{da}{dt} = \frac{d}{\lambda} L \frac{\dot{\lambda}}{gbs} \]  

(9)

LONG CRACK GROWTH

Long crack growth can be modeled by linear elastic fracture mechanics using the Paris law representation for a surface crack subjected to a constant stress cycle:

\[ \frac{da}{dN} = C(\Delta K)^n \]  

(10)

Crack closure effects can be accounted for by modifying the stress intensity range. Dwell period can have a considerable effect on long crack growth. Competing effects of material weakening through oxide ingress at the crack tip and oxide induced closure effects can change the long crack growth rate. In most cases the change in rate of crack growth at elevated temperatures can be modeled by modifying the Paris law coefficient [15].

MATERIAL MODEL INPUT PARAMETERS

The material under investigation is a Ni-base superalloy. The material model properties that were input for the simulation include the statistical distributions and the averages and their coefficient of variation (COV) for random variables. Some of the material property variables are temperature sensitive. So it is necessary to have distinct material models for different temperatures. Some of the inputs for 1200ºF are as follows:

- Average grain size = 0.00125 in
- Grain size COV = 0.05
- Average frictional strength = 95 Ksi
- Frictional Strength COV = 0.18
- Average defect size = 0.001 in
- Defect size COV = 0.05
- Paris law exponent = 3.336
- Average Paris coefficient = 2.45e-10
- Paris coefficient COV = 0.3

RESULTS AND DISCUSSION

SMOOTH ROUND BARS

Smooth round bar (SRB) simulation without dwell was conducted to establish a baseline to study the effect of dwell on life. Fifty SRBs were simulation using Monte Carlo techniques to obtain a sizeable statistical sample. The statistical distribution plot for all three individual stages along with total life distribution for a maximum applied stress of 140 Ksi and an R ratio of 0.05 is plotted in Figure 1. At the lower end of total life sample set, none of the three individual damage stages dominated in terms of number of cycles. However, it can be seen that at the higher end of the statistical sample, the short crack stage has a more significant effect on the total life. Long crack growth has the least effect on total life.

The cumulative life distribution plot for fatigue cycling without dwell is shown in Figure 2. The cumulative life at all levels of reliability is dominated by nucleation. There is very little difference in the cumulative life at the end of short crack growth stage and the long crack growth stage. This indicates that in the absence of dwell, the long crack growth stage does not have a significant contribution to the total life.

The simulation results for SRBs with dwell for the same maximum applied stress and R ratio as the fatigue only case and a 120 second dwell period is shown in Figure 4. The addition of the dwell period resulted in most of the cracks originating from inclusions. As a result few transgranular nucleated cracks are seen and even that only at higher end of total life samples. In this case also short crack stage is the dominant stage in terms of number of cycles, especially for samples with high life. In the samples with the lowest lives, the short crack and long crack lives are comparable. Comparison of total life in Figure 3 for fatigue cycling with dwell and total life in Figure 1 for pure fatigue cycling shows the deleterious effect of the 120 second dwell period on the total life distribution. There is almost an order of magnitude of difference between the two cases.

The cumulative life distribution plot for fatigue cycling with 120 second dwell is shown in Figure 3. It can be seen that above a probability of failure of 0.8, there is practically no contribution from long crack growth to the cumulative life of the SRB. For the lower life specimens, both short crack and the long crack stages are
Figure 1: Simulation results for a SRB for a maximum stress of 140 Ksi and \( R = 0.05 \) at 1200ºF.

Comparable and in fact long crack growth has a bigger contribution.

Figure 2: Cumulative life distribution at the end of each stage for a SRB without dwell.
Figure 3: Simulation results for a smooth bar for a maximum stress of 140 Ksi and R=0.05 and hold time = 120 seconds at 1200°F.

Figure 4: Cumulative life distribution at end of each stage for a smooth bar with dwell
Simulations were also conducted for the same material at 1350°F. The simulated results were compared with actual experimental results to validate the simulation. The results, plotted in terms of relative stresses and relative life, are shown in Figure 5. The simulated data matches very well with the available test data points for both temperatures at various stress levels.

**NOTCHED SPECIMENS**

Finite element models of the notched specimens geometry were created using ANSYS. Elastic-plastic analysis along with creep analysis was conducted to estimate the stress at the notch as a function of time. Taking advantage of the symmetry, a quarter section of

![Figure 6: Initial equivalent stress contour for a DEN specimen at maximum load at 1200°F](image)
a double edge notched model was analyzed. The initial equivalent stress contour for maximum concentrated elastic stress of 218 Ksi is shown in Figure 6. The effect of tensile dwell on the stress field at the notch was studied by loading the specimen at maximum stress for 91 hours. The equivalent stress contour at the end of this period is shown in Figure 8. Comparing the two figures, it is seen that the maximum stress at the notch has dropped from 134.4 Ksi to 124.5 Ksi after 91 hours. This can have a significant effect on the predicted life of the notched component under strain controlled conditions.

Double edge notched specimen stress results were used to simulate the life of a notched specimen of the same material. A maximum stress level was chosen where four experimental test data points were available for comparison. This gives a better idea of the variation of the actual life data at that stress level and can be compared to the simulated test data. The results of the simulation are plotted in terms of relative stresses and relative life in Figure 7. The comparison shows that the
50 simulated data matches very well with the four actual test data and covers the scatter in actual life very well.

**CONCLUSION**

A physics-based simulation model was created to predict the creep fatigue reliability of engineering alloys at elevated temperatures. Most of the input variables are experimentally measurable. The simulations predict the importance of estimating the life spent at the crack initiation stage. There is a substantial debit, by factor of almost 10, in the short crack stage due to addition of the dwell period. Addition of dwell period also predicted that most of the specimens to fail from inclusions and defects. The overall effect of damage during the dwell period on the reliability of the specimen is quite significant.

The predicted results compared favorably to actual test results for both the smooth round bars as well as the notched specimens. The models allow the structural engineer to systematically and quantitatively assess the influence of the material, loading and temperature on the overall reliability of the structure. The models are used to identify the sources of uncertainty and quantify the sensitivity of the reliability to the uncertainty.

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**REFERENCES**